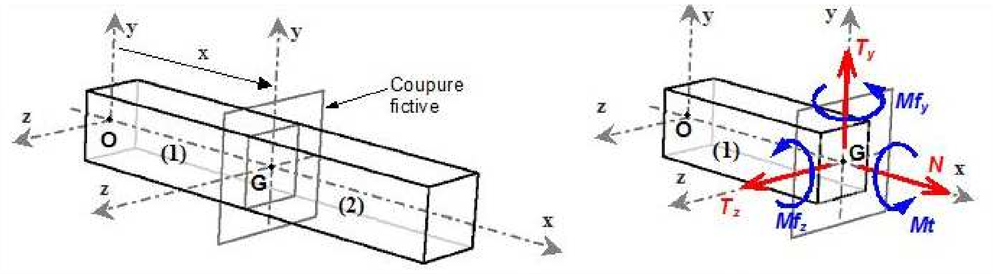
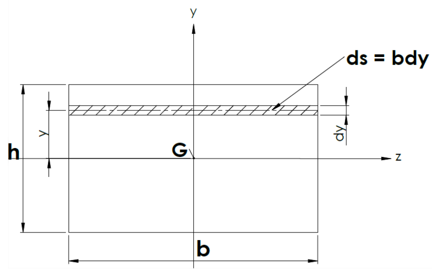
Modèle poutre soumis à sollicitations:



<u>**Définition: Moment quadratique par rapport à l'axe GZ**</u>: ➔ $I_{GZ} = \int_S y^2 \mathrm{ds}$



- <u>**Poutre à section rectangulaire:**</u>

<u>**Premier calcul:**</u>

$I_{GZ} = \int_S y^2 \mathrm{ds} = \int_S y^2\, b\mathrm{dy}$

La primitive de y^2 est $\frac{y^3}{3}$

$I_{GZ} = b \int_S y^2 \mathrm{dy} = \mathrm{b}\left[\frac{y^3}{3}\right]_{\frac{-h}{2}}^{\frac{+h}{2}} = \mathrm{b}\left[\frac{\frac{h^3}{8}}{3} + \frac{\frac{h^3}{8}}{3}\right] = b\left[\frac{h^3}{24} + \frac{h^3}{24}\right] = \frac{bh^3}{12}$ ➔ $I_{GZ} = \frac{bh^3}{12}$

<u>**Second calcul:**</u>

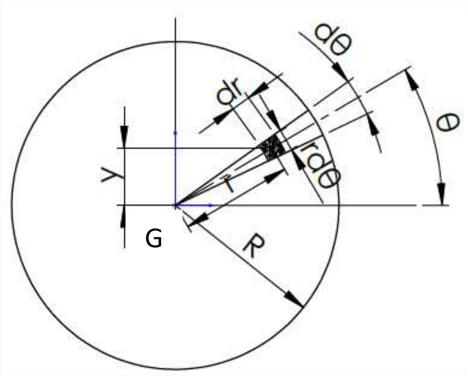
$I_{GZ} = \iint_S y^2 dzdy = \int_{\frac{-h}{2}}^{\frac{+h}{2}} y^2 dy \times \iint_{\frac{-b}{2}}^{\frac{+b}{2}} dz = \left[\frac{y^3}{3}\right]_{\frac{-h}{2}}^{\frac{+h}{2}} \times [z]_{\frac{-b}{2}}^{\frac{+b}{2}} = \left[\frac{h^3}{2^3\times 3}\right] - \left[\frac{-h^3}{2^3\times 3}\right] \times \frac{b}{2} - \left(\frac{-b}{2}\right) = \left[\frac{2h^3}{2^3\times 3}\right] \times \frac{2b}{2} = \left[\frac{h^3}{2^2\times 3}\right] \times b$

- <u>**Poutre à section carrée de cotée "a" :**</u>

a= b =h

$I_{GZ} = a \int_S y^2 \mathrm{dy} = \mathrm{a}\left[\frac{y^3}{3}\right]_{\frac{-a}{2}}^{\frac{+a}{2}} = \mathrm{a}\left[\frac{\frac{a^3}{8}}{3} + \frac{\frac{a^3}{8}}{3}\right] = a\left[\frac{a^3}{24} + \frac{a^3}{24}\right] = \frac{2a^4}{24}$ ➔ $I_{GZ} = \frac{a^4}{12}$

- **Poutre à section circulaire :** ➔ $I_{GZ} = \int_S y^2 ds$



l'élément de surface ds = dθrdr et y = r sinθ

$I_{GZ} = \iint_S y^2 d\theta r dr = \iint_S r^2 sin^2\theta d\theta r dr = \iint_S r^3 sin^2\theta d\theta dr$

on a : $sin^2\theta = \frac{1-cos2\theta}{2}$

$I_{GZ} = \iint_S r^3 \left(\frac{1-cos2\theta}{2}\right) d\theta dr = \frac{1}{2}\int_0^R r^3\, dr \times \int_0^{2\pi}(1-cos2\theta)\, dr$

La primitive de $r^3 est$ $\frac{r^4}{4}$ et la primitive de 1-cos2θ est θ- $\frac{1}{2}\sin 2\theta$

$I_{GZ} = \frac{1}{2}\left[\frac{r^4}{4}\right]_0^R \times \left[\theta - \frac{1}{2}sin\theta\right]_0^{2\pi} = \frac{1}{2}\left[\frac{R^4}{4}\right] \times [2\pi] = \frac{\pi R^4}{4}$

$R = \frac{D}{2}$ ➔ $I_{GZ} = \frac{\pi}{4} \times \left(\frac{D}{2}\right)^4$ ➔ $I_{GZ} = \frac{\pi D^4}{64}$